FORCES SET UP IN A CONDUCTOR
BY A
CURRENT "PINCH EFFECT"

G. M. VALERIO

ARMOUR INSTITUTE OF TECHNOLOGY

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Valerio, Giuseppe Mastro
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FORCES SET UP IN A
CONDUCTOR BY A CURRENT

"PINCH EFFECT"

A THESIS

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GIUSEPPE MASTRO-VALERIO

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Approved
[illegible] Freeman

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1.

The object of this Thesis was:- "To find the relation which the distance from the center of a conductor has on the so called "pinch effect" in it". This however, I was unable to find out experimentally. In spite of the very valuable help which I received from Mr. Wilcox (and I thank him very much for that), still I could not get an apparatus which at least could show that the pinch effect is different at different parts of the conductor, even if I could not measure such difference. I actually did carry out one experiment which was suggested by Mr. Rodtke, but this did not prove of any value. There are very little or no references from which to get any ideas to work with on this subject. In fact, the only reference is an article published in the "Physical Review" of Mar. 1907 - page 474, written by Mr. Edwin F. Northrup. This however, is an entirely theoretical article, and although it describes experiments which prove that such a thing as the pinch effect exists, still it does not give any idea as to how to go about to prove the relation of the pinch effect to the distance from the center of the conductor. Before I go any further, however, I think it best to say something about this, so-called "pinch effect".

II

If A and B (Fig.-I) are two conductors each carrying a certain current in the direction indicated by the arrows, fields will be set up

around them as indicated by the circles. If the lines of force of one unite with the line of force of the other, the two will combine and as these lines act as stretched rubber bands, the result will be that one conductor will tend to move toward the other. Now suppose we consider a large conductor composed of an infinite number of small conductors with infinitesimal cross sections. When the above conductor carries a current each one of these conductors will carry a current in the same direction, so that there will be an attraction toward each other with the result that they will all tend to move toward the center, and actual pressures will be set up in the conductor. This pressure is what is usually called "pinch effect". From the above considerations it is seen that this pressure depends on the field set up by the current going through the conductor, it must be maximum at the center and zero at the surface of the conductor.

III

Let (Fig. 2) represent a cross section of a conductor of a [illegible]rent length and removed from other circuits. Let the above cross section be divided into a large number of annular spaces of the radial width equal to dr. Let r be the distance from the central axis to any point within the cross section; R equals the radius of conductor.

If we consider the action of the magnetic field on all parts of the conductor carrying current, we have that the intensity of the magnetic field at a distance r is

$$I_r = \frac{2Ir}{R^2}$$

I equal total current in conductor.

If we call A total area of conductor, da area of any annular space, and di current carried by it, we have $\frac{I}{A} = \frac{di}{da}$

$$da = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr$$

where we neglect $\overline{dr}^2$

As $\pi R^2 =$ total area

$$di = \frac{2Ir\,dr}{R^2}$$

Whether the current I is flowing upward or downward in the above conductor is immaterial, because although the direction of the magnetic lines of force would be reversed, still the current elements would tend to move toward the axis. In electromagnetic units the force in dynes with which a length (l) of any conductor is acted upon to move at right angles to the lines of magnetic force is (Maxwell Vol. II Par. 490) numerically equal to the product of the length l, to the strength of the current in the conductor and to the field intensity where the conductor is located. If we call this force dF we have

$$dF = l\,di\,T_r$$

Let dg equal the force intensity acting radially inward.

$$dg = \frac{dF}{2\pi r l} = \frac{2I^2 r\,dr}{\pi R^4}$$

The sum of the force intensities due to the current in all the annular spaces, which lie, in the space included between R and radius r is

$$g = \frac{2I^2}{\pi R^4}\int_r^R r\,dr = \frac{I^2}{\pi R^4}\left(R^2 - r^2\right)$$

If we let I^1 equals current density in the conductor we have

$$I^2 = \pi^2 I'^2 R^4$$

substituting

$$g = \pi I'^2 (R^2 - r^2)$$

When r equals o

$$g = \pi I'^2 R^2$$

which is the pressure at the center of the conductor and where it is maximum.

If r equals R then

g equals o

which is the pressure at the surface of the conductor.

If r equals 1/2 R then

$$g = \pi I'^2 \left(R^2 + \frac{1}{4}R^2\right) = \frac{3}{4}\pi I'^2 R^2$$

From these three values it is seen that the curve should be a straight line. Fig. 3 shows a set of assumed values and the curve obtained from them.

The total pressure on the surface of a cylinder is the intensity of the force applied on it, times the total area of the surface of the cylinder.

$$P = 2g\ell r = 2\pi \ell r I'^2 (R^2 + r^2)$$

If we consider the fact of the mutual attractions between two conductors carrying current in the same direction and start from the formula

$$F = 2ii_1 \frac{\ell}{r}$$

where F equals the force, i and i the current carried by each conductor, l the length considered, and one conductor to be a cylinder with radius r and outside radius r plus dr.

Again we have

$$i = \frac{r^2}{R^2} I$$

and if $2\pi rdr$ is the cross section of the annular area

$$di = \frac{2r\,dr\,I}{R^2}$$

df is the pressure over the surface of the cylinder of radius r produced by the currents i and di and

$$dF = \frac{2i\,di'\ell}{r} = \frac{4\ell I^2 r^2 dr}{R^4}$$

and the pressure per unit of area

$$dg = \frac{dF}{2\pi r\ell} = \frac{2I^2 r\,dr}{\pi R^4}$$

integrating

$$g = \frac{2I^2}{\pi R^4}\int_r^R r\,dr = \frac{I^2(R^2 - r^2)}{\pi R^4}$$

$$P = 2\ell I^2 r \frac{(R^2 - r^2)}{R^4}$$

Which are the same expressions as we obtained from the other method.

Now if we let l equal 1 in the formula

$$F = \frac{2 i_1 i l}{r}$$

we have

$$F = \frac{2 i i_1}{r}$$

the current carried by a conductor of radius r is

$$i' = \frac{r^2}{R^2} I$$

substituting we have

$$F = \frac{2 r i_1 I}{R^2}$$

If I_1 equal current density in the conductor and i equal 1, then

$$F_t = 2 \pi I_1 r$$

which is the attraction in the interior of a conductor carrying current.

IV

The above is entirely theoretical and although it can be shown that it is absolutely correct as far as theory and mathematics is concerned still nobody has yet experimentally proved that the forces act as proved above and as expressed in the equations.

Of course experiments have been conducted which proved that there are such forces but what their magnitude is at different distances from the center of the conductor is yet unknown.

IV-A

One apparatus which proves that there is such pressure, is the "Mercury interrupter", which was first built by Mr. Hering. Fig. 4 shows this interrupter. Two square holes were cut in a thick sheet of hard rubber. A little channel about 3/4" deep and about 1/4" wide and 1/2" long joined these two holes. When a current of 100 amp. was passed the liquid tended to move toward the two holes, and when the current was increased the depression would reach the bottom of the channel and break the circuit; as soon as the circuit was broken the mercury would come together, completing the circuit again

the depression would take place breaking the circuit and so the interruption would go on.

IV-b

Another experiment was then devised and an apparatus as shown in Fig. 5 was constructed. This consisted of a wooden box formed of two compartments, connected by a deek channel. One side of the box was made of mica so as to be able to see from the outside to the level of the liquid; electrodes were fastened at each end of the box. A liquid alloy of potassium and sodium was poured in this box to a depth of two inches, the remaining space was filled with kerosene. When a current of about 180 amp. was passed a depression of 1-3/4" was noticed. In this experiment it was found that a given current gave a given constant depression.

IV-c

By means of the apparatus in Fig. 6 not only were they able to prove the fact that the pinch effect existed but they actually measured the pressure at the center of the conductor.

As it can be seen from Fig. 6 the apparatus consists of two tubes made of non-conducting material, the electrodes being attached to the inside tube and the two tubes being interconnected by the holes h. S is a ring put there so to further reduce the cross section of the inner tube. On the top there is a hole H. When no current was passed through, the mercury was at a certain level, but when a current was passed the mercury rose in the hole H, so that with 1300 amp. a rapid stream was obtained. When a tube was fitted in H, it was found that the mercury rose.3 of an inch.

In order to increase this effect they fitted in place of S a disk made of iron and fiber as seen in Fig. 7. This disk consists of a ring of fiber like S and a piece of iron, then a slot was cut into this disk and the whole thing was fitted in the tube so that the end of the slot registered with one of the holes h. (Fig. 6) Now the intensity of the magnetic field in the

space of the narrow slot should be greatly increased by the presence of the iron; at the same time the conducting mercury filaments in the slot should be urged with increased force toward the axis with a resulting greater hydrostatic pressure at the axis. With this scheme they obtained about the same effect with [illegible] amp. as they obtained before with [illegible] amp.

Mercury ammeters are built on the same principle of this apparatus. The only difference is that the different pressures in a series of cups are added in series as shown in the Fig. 8. With this scheme, the high pressure at the middle of cup A is added at the edge of cup B, where the pressure is the least, and these two pressures are added to the edge of cup C, etc. Now a very little current will cause considerable rise of the mercury. In a mercury ammeter, however, instead of making mercury, which is very heavy, rise in the glass tube at the top, they make the mercury push up some oil or colored water, which is much lighter and therefore it takes less pressure to push it up.

V.

Now the question arises. Is it possible to measure the relation of the rise to the distance from the center of the conductor? Of course it is easy to calculate it, but can we actually measure it experimentally? At first I thought of the scheme shown in Fig. 9, which is a cup of insulating material with its top and bottom made of copper so to attach the electrodes to them. A and B are two glass plates. In passing the current, I thought at first that the liquid would rise most in the center, where according to the theory the pressure is the highest and in that way get a curve which would be maximum at the center and zero at the sides. But soon I realized that this would not work, because of the fact that the pressure in the liquid would be equal at all points (according to the well known law of physics) and therefore if the liquid rose at all, it would rise equally at all points. So that I soon disregarded this plan.

Then Dr. Redthe suggested that I take a conductor with a very large corss section and drill holesaxially through it. Fill these holes with mercury and in passing the current through the conductor the mercury should rise at different heights, if the holes were drilled at different distances from the center. I thought this apparatus would not work for several reasons, I nevertheless did build it up.

VI

I took (Fig. 10) a conductor whose diameter was 4" and [illegible] it I drove four or five axial holes all at different distances from the center of the conductor. Then I drove other holes radially to the conductor and connecting the axial holes to the outside surface of the conductor. Now I closely fitted a piece of glass to the top of the axial holes, and to the radial holes I fitted a glass tube bent at right angle. In this manner I built a U tube one branch of which was the axial hole in the conductor. First I put mercury in these U tubes and passed a current. I was not able to notice any rise with a current of 2000 amp. going through it. Then I thought that the mercury might be too heavy for sush an experiment, so I tried the same experiment with the same current, first with salt water and then with a solution of copper sulphate, and still no results.

VII

As I said before, there are several reasons why this scheme would not work. First of all it is out of the question that the pinch effect in the iron itself would cause the liquid to rise, because if there is such a thing as a pinch effect, it is not high enough to make cold iron contract or expand, but it is taken in strains in the iron itself, so that the cross section of the hole will not change. But just for the sake of argument let us suppose for a minute that the liquid is actually pinched. I tried this experiment first with both sides of the U tube open to the atmosphere. When no current passes and therefore there is no pinch effect the liquid is at the same level in the two branches and the

only pressurein them is the atmospheric pressure (14.4 lbs./sq.in.) which is the same in the two branches and therefore its summation is zero. Now pass the current; the pinch effect pushes the liquid upward. The minute it does so a vacuum must be created in the U tube in order to allow the liquid to move, but if this vacuum is created then the atmospheric pressure will immediately come into play and keep the liquid at the same level, the pinch effect not being as strong as the atmospheric pressure. Now I closed the metallic branch of the U tube and I passed a current. Now again the liquid will be pushed toward the open branch and again the tendency to create vacuum, with the result that there will be no rise of the liquid due to the atmospheric pressure. From this we can see that the pinch effect in the iron itself will not cause the liquid ro rise.

The next thing to do is to think that the pinch effect in the liquid itself might cause it to rise , this however gets us into other difficulties. Of course I know that I had about 2000 amp. going through the piece of iron, but how was the current distributed in that mass of iron? We know theoretically how the field was distributed and hence the pinch effect, but what part of the current for instance passed at half the distance between the center and the outside surface of the iron. And what is more to the point, does as much current pass through the liquid as passes through the iron immediately surrounding the liquid? We can see at once that,that current passing through the liquid is much less than that passing through the iron. Because the conductivity of the iron is much higher. But then if the conductivity of the iron is higher its impedance (A.C. having been used) is also higher while that of the liquid is practically zero, but even so,still the current passing through the liquid is much less than 2000 amp. According to Dr. Northrup's article the pinch effect is noticeable only at high currents so that perhaps the reason why we were unable to get results in this experiment was that not enough current went through

the liquid. Then Mr. Wilcox suggested that I build the apparatus shown in Fig. 6 in the block of iron, as shown in Fig. 11, at different distances from the center of the conductor. Have the mercury up to a certain height in the tube A and then pass a current. Of course it is certain that the mercury would rise in this case, if the current is high enough, but then we would be getting away from our original proposition viz: "To find the relation of the pinch effect to the distance from the center of a conductor. In this case we would be proving that the pinch effect is diferent with different currents, because we would not be measuring the pinch effect in the conductor itself, but the pinch effect in a column of mercury put at certain distance from the center of the conductor. In which case we might just as well build an apparatus as we have in Fig. 6 and just pass different currents through it without building it in a mass of iron as was suggested.

VIII

I am sorry I am unable to show better results than I have actually obtained in this work, I must point out however that I did the best I could if we consider that although I have had plenty of good, hard experimental work in the schook still I had never before done research work. To make things still worse, very little or no work has been done on this line before and therefore there are no references to be found, which might give at least some suggestions of how to go at this work. Before I close however I must again thank Mr. Wilcox for his most valuable help which he offered.

FINIS.

FIGURES

Fig. 1

Fig. 2

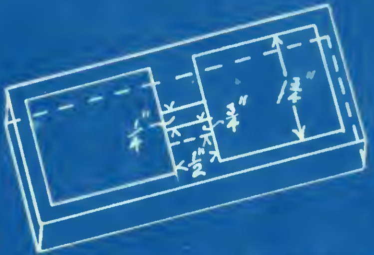


Fig 4. – Mercury Interrupter.

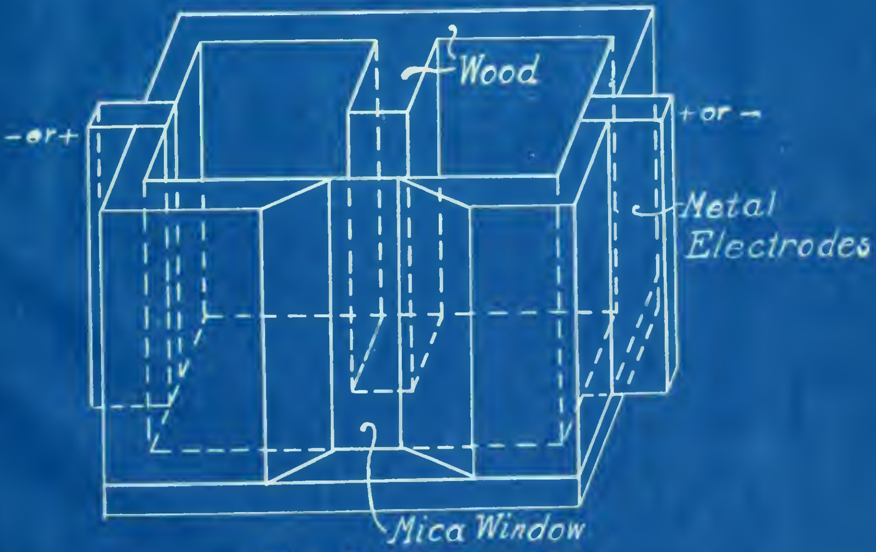
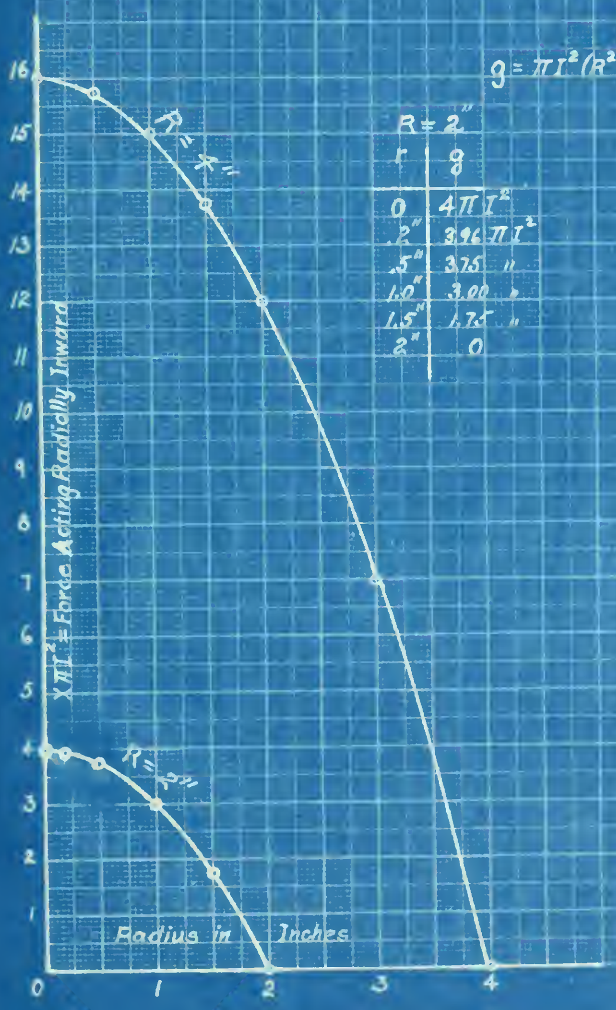


Fig. 5

Fig. 3



$g = \pi I^2 (R^2 - r^2)$

R = 2"

r	g
0	$4 \pi I^2$
.2"	$3.96 \pi I^2$
.5"	3.75 〃
1.0"	3.00 〃
1.5"	1.75 〃
2"	0

R = 4"

r	g
0	$16 \pi I^2$
.5"	15.75 〃
1.0"	15.0 〃
1.5"	13.75 〃
2	12.0 〃
3	7.0 〃
4	0 〃

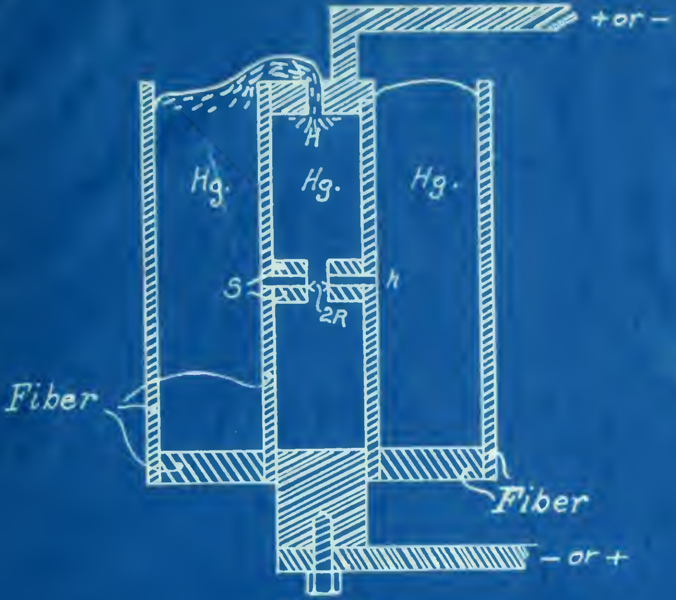


Fig. 6

Fig. 7

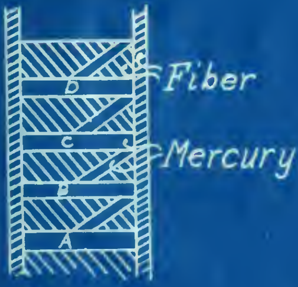


Fig. 8

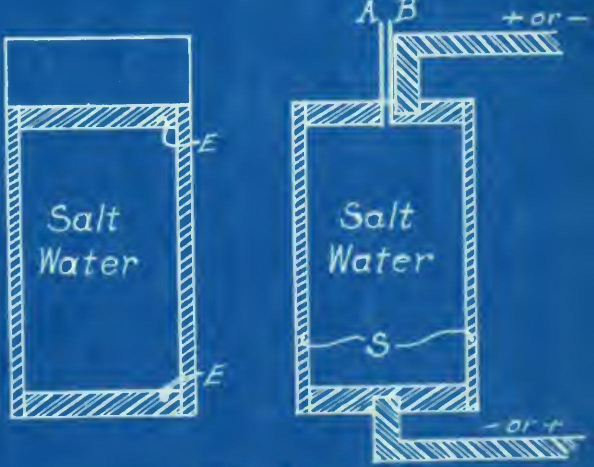


Fig. 9

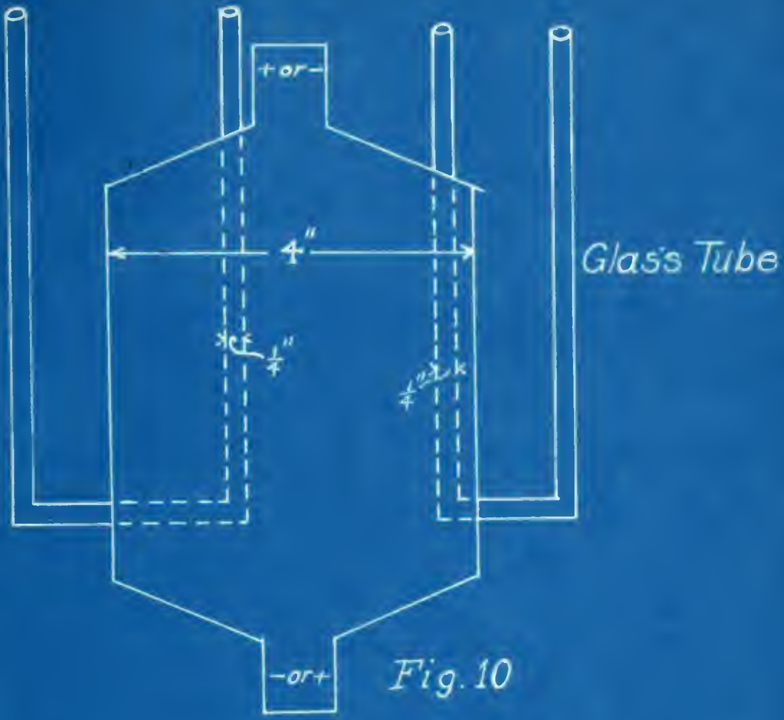


Fig. 10

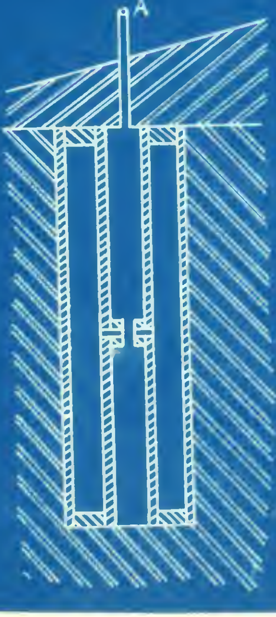


Fig. 11